

$$E\{x(t)x(t+\tau)\}$$

$$= E\left\{\underbrace{\left(\sum_{k=-\infty}^{\infty} a_k p_T(t-kT)\right)}_{x(t)} \times \underbrace{\left(\sum_{m=-\infty}^{\infty} a_m p_T(t+\tau-mT)\right)}_{x(t+\tau)}\right\}$$

$$= E\left\{\sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_k a_m \cdot p_T(t-kT) \times p_T(t+\tau-mT)\right\}$$

Take expectation operator inside

$$= \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underbrace{E\{a_k a_m\}}_{\text{if } k \neq m} p_T(t-kT) \times p_T(t+\tau-mT)$$

$$E\{a_k a_m\} = \underbrace{E\{a_k\} E\{a_m\}}_0$$

if $k \neq m$

$= 0$
since symbols are zero mean IID.

$$= \sum_{k=-\infty}^{\infty} \frac{E\{a_k^2\} p(t-kT) p(t+\tau-kT)}{A^2 = P_d} \quad \text{Data Symbol Power}$$

$$Pr(a_k = A) = \frac{1}{2}$$

$$Pr(a_k = -A) = \frac{1}{2}$$

$$E\{a_k^2\} = \frac{1}{2} \cdot A^2 + \frac{1}{2} (-A)^2 = A^2$$

$$= P_d \sum_{k=-\infty}^{\infty} p_T(t-kT) p_T(t+\tau-kT)$$

Depends on t

$$E\{x(t)x(t+\tau)\}$$

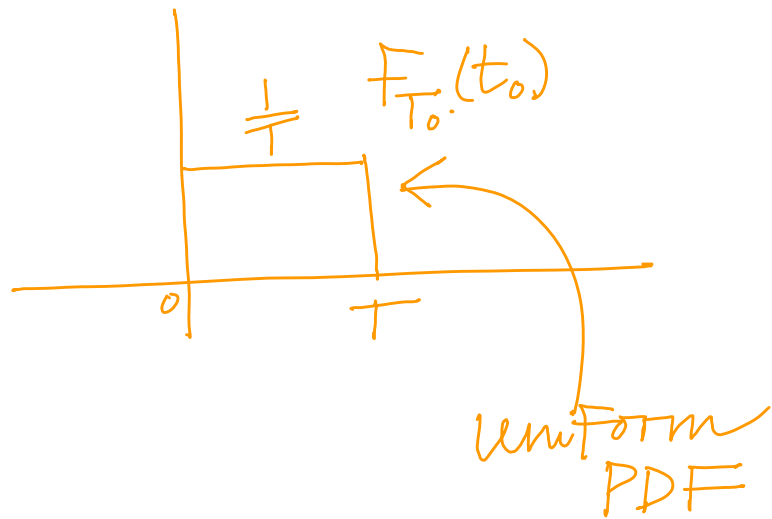
Hence NOT WSS

Wide Sense Stationary

$$x(t) = \sum_{k=-\infty}^{\infty} a_k p_T(t-kT-t_0)$$

random Delay

$t_0 = \text{Random Delay}$
 Uniformly Distributed
 in $[0, T]$



$$E\{x(t)x(t+\tau)\}$$

$$= P_d \sum_{k=-\infty}^{\infty} E\{P_T(t-kT-t_0) \times P_T(t+\tau-kT-t_0)\}$$

Average or
 Expected value wrto
 t_0 .

$$= P_d \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} f_{T_0}(t_0) P_T(t - kT - t_0) \times P_T(t + \tau - kT - t_0) \times dt_0.$$

Averaging wr to t_0
random Delay

$$f_{T_0}(t_0) = \begin{cases} \frac{1}{T} & 0 < t_0 \leq T \\ 0 & \text{otherwise} \end{cases}$$

$$= \frac{P_d}{T} \sum_{k=-\infty}^{\infty} \int_0^T P_T(t - kT - t_0) \times P_T(t + \tau - kT - t_0) \times dt_0.$$

$$t_0 + kT = \tilde{t}_0 \quad dt_0 = d\tilde{t}_0$$

$$= \frac{P_d}{T} \sum_{k=-\infty}^{\infty} \int_{kT}^{(k+1)T} p(t - \tilde{t}_0) p(t + \tau - \tilde{t}_0) d\tilde{t}_0$$

each integral from kT to $(k+1)T$

R from $-\infty$ to ∞

$$= \frac{P_d}{T} \int_{-\infty}^{\infty} P_T(t - \tilde{t}_0) P_T(t + \tau - \tilde{t}_0) d\tilde{t}_0$$

$$t + \tau - \tilde{t}_0 = t'_0$$
$$-d\tilde{t}_0 = dt'_0$$

$$= \frac{P_d}{T} \int_{\infty}^{-\infty} P_T(t'_0 - \tau) P_T(t'_0) (-dt'_0)$$

reverses order of integration

$$= \frac{P_d}{T} \int_{-\infty}^{\infty} P_T(t'_0) P_T(t'_0 - \tau) dt'_0$$

Does NOT depend on t
only depends on τ
Hence, $x(t)$ is WSS

$$E\{x(t)x(t+\tau)\} = R_{xx}(\tau).$$

$$\int_{-\infty}^{\infty} P_T(t'_0) P_T(t'_0 - \tau) dt'_0$$

$$R_{P_T P_T}(\tau)$$

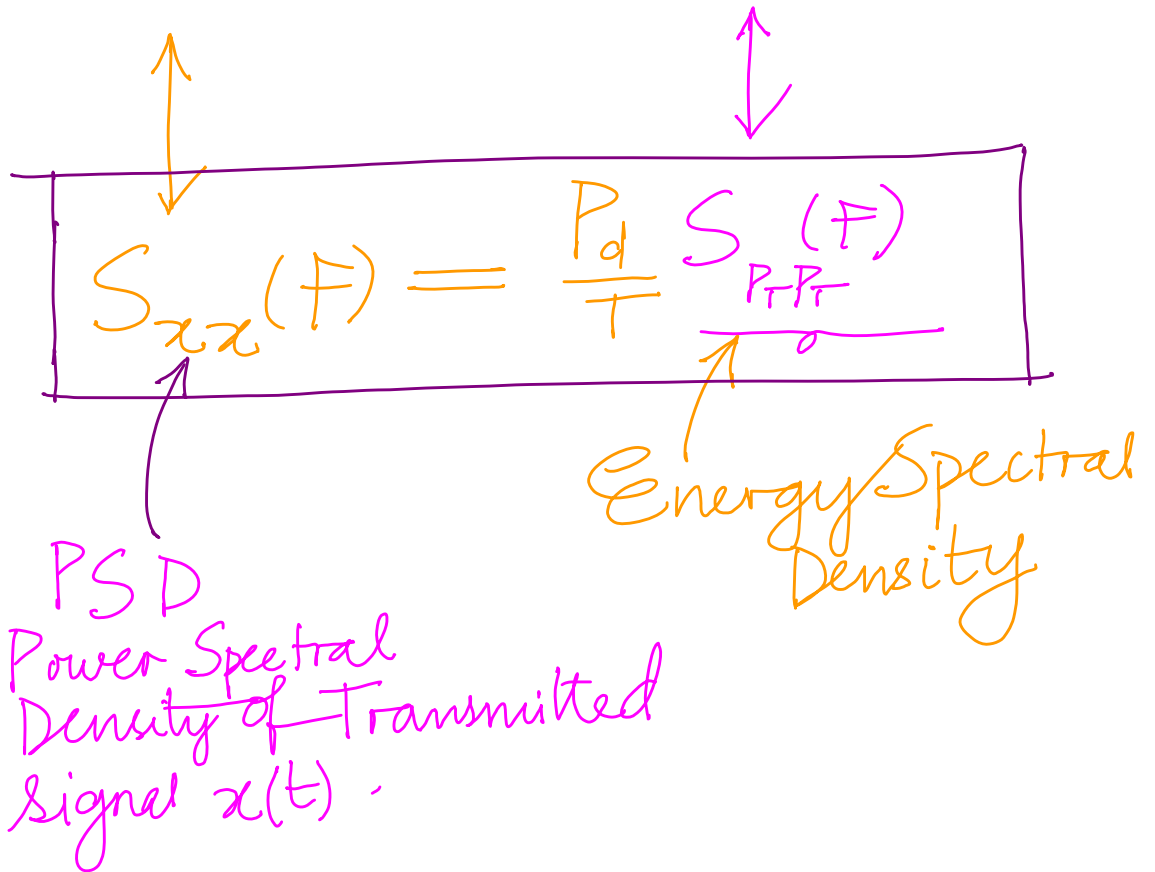
Auto correlation
of pulse $P_T(t)$

$$R_{xx}(\tau) = E\{x(t)x(t+\tau)\}$$

$$= \frac{P_d}{T} R_{P_T P_T}(\tau)$$

Autocorrelation of
signal depends on
auto-correlation of
pulse $P_T(t)$.

$$R_{xx}(z) = \frac{P_d}{T} \cdot R_{PP}(z)$$



$$S_{xx}(f) = \frac{P_d}{T} \frac{S_{PP}(f)}{0}$$

PSD
Power Spectral
Density of Transmitted
Signal $x(t)$.

Energy Spectral
Density